QUIZ 16 SOLUTIONS: LESSONS 20-21
OCTOBER 19, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Compute the second order partial derivatives of

$$f(x, y) = \ln(x^2 y).$$

Solution: We begin by finding the first order partial derivatives:

$$\begin{aligned}
f_x &= \frac{\partial}{\partial x}(\ln(x^2y)) \\
&= \frac{\frac{\partial}{\partial x}(x^2y)}{x^2y} \\
&= \frac{y\frac{\partial}{\partial x}(x^2)}{x^2y} \\
&= \frac{y(2x)}{x^2y} \\
&= \frac{2}{x} \\
f_y &= \frac{\partial}{\partial y}(\ln(x^2y)) \\
&= \frac{\frac{\partial}{\partial y}(x^2y)}{x^2y} \\
&= \frac{x^2\frac{\partial}{\partial y}(y)}{x^2y} \\
&= \frac{x^2}{x^2y} \\
&= \frac{1}{y}
\end{aligned}$$

Next, we find the second order partial derivatives:

$$\begin{aligned} f_{xx} &= \frac{\partial}{\partial x}\underbrace{\left(\frac{2}{x}\right)}_{f_x} \\ &= \boxed{-\frac{2}{x^2}} \\ f_{xy} &= \frac{\partial}{\partial y}\underbrace{\left(\frac{2}{x}\right)}_{f_x} \\ &= \boxed{0} \\ f_{yy} &= \frac{\partial}{\partial y}\underbrace{\left(\frac{1}{y}\right)}_{f_y} \\ &= \boxed{-\frac{1}{y^2}} \end{aligned}$$

2. [5 pts] Use increments to estimate the change in z at $(5, 9)$ if

$$\frac{\partial z}{\partial x} = -9x - 10, \quad \frac{\partial z}{\partial y} = 6y + 8$$

and the change is x is .6 and the change in y is .8. Round your answer to the nearest tenth.

<u>Solution</u>: The incremental approximation formula is

$$\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y.$$

We write

$$\frac{\partial z}{\partial x}(5, 9) = -9(5) - 10 = -55$$

and

$$\frac{\partial z}{\partial y}(5, 9) = 6(9) + 8 = 62.$$

Further, we know $\Delta x = .6$ and $\Delta y = .8$. Thus, by our formula,

$$\begin{aligned} \Delta z &= \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y \\ &= (-54)(.6) + 62(.8) \\ &= \boxed{16.6} \end{aligned}$$